INTEGRATED VEHICLE ROUTING PROBLEM WITH EXPLICIT CONSIDERATION OF SOCIAL COSTS IN HUMANITARIAN LOGISTICS

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ABSTRACT

A review of the literature indicates that most formulations for humanitarian logistics are extensions of methods for commercial applications that do not fully account for the particular challenges in humanitarian operations. In contrast to commercial logistics, operational costs are of secondary importance for humanitarian logistics since responders are frequently more concerned about allocating their limited resources in such a way that they maximize the benefits to the affected population. In this context, human welfare at the sites impacted by extreme events can be considered as an externality cost derived from the distribution strategy. This research considers the loss in welfare associated with victims not having access to critical commodities as a social cost that should be priced and incorporated to the decision making process. Following this approach to humanitarian logistics modeling, this paper presents formulations for vehicle routing in which the optimal solution is the strategy minimizing the total costs incurred in responding to the emergency, as determined by both operational and social considerations. These formulations provide a new and integrated perspective to humanitarian logistics because they recognize the significant impacts that logistical decisions may have on the victims’ wellbeing and vice versa.

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1. INTRODUCTION

Recent disasters such as the 2001 terrorist attack to the World Trade Center, the 2004 Indian Tsunami, the 2005 Hurricane Katrina, and the 2010 Haitian Earthquake have evidenced, sometimes tragically, the challenges that extreme events pose to humanitarian logistics. In situations like these, critical resources (e.g., water, food and medical supplies) available in the affected areas are usually either destroyed by the event itself or quickly depleted during the immediate response, hence requiring the rapid and efficient deployment of external assistance to reduce the loss of human life and property. However, supplying goods to the affected areas is usually a very difficult task because of the damages to the physical and virtual infrastructures and the very limited or non-existent transportation capacity. This situation is aggravated by the fact that many humanitarian supply chains form and operate in an ad hoc basis, which prevents them from developing the technical expertise and the scientific methods to properly analyze and coordinate the flows of goods under such extreme conditions. This reactive approach introduces large inefficiencies in the system and leads to poor levels of service to the affected areas, reflected in delayed or incorrect deliveries, redundant activities and unmet needs.

Although emergency and commercial supply chains have the same general purpose—delivering goods to points of demand—they differ significantly in several key aspects such as, e.g., the objectives pursued, the characteristics of the commodity flows, the decision making structures and the state of the supporting systems. However, as discussed in the next section, many analytical formulations in the literature for humanitarian logistic do not consider these differences and extend to emergency operations the models developed for traditional supply chain problems focused on the minimization of operational costs. This objective may be appropriate for commercial activities, but minimizing operational costs in the aftermath of large-scale disasters is an issue of secondary importance for responders frequently unable to meet all emerging needs and primarily concerned about allocating their limited resources in such a way that they maximize the benefits to the affected population. This situation presents a fundamental dilemma for humanitarian logistics: how to harmonize these two seemingly disparate objectives? Maintaining high levels of service to increase wellbeing at the affected areas requires larger expenditures on supplies, personnel, transportation and other resources, which is not always a financially feasible option. On the other hand, aggressive measures to reduce operational costs could negatively affect the victims’ welfare to the extent that many lives could be lost in extreme cases. In this context, human welfare at the impacted areas can be considered as an externality cost stemming from the distribution strategy and, as such, it should be priced and incorporated to the decision making process, a concept initially proposed by Holguin-Veras et al. Under this approach to humanitarian logistics, the optimal solution to the distribution problem is the strategy minimizing the total costs incurred in emergency response, both operational and social.

Incorporating social costs to the models for humanitarian logistics presents several challenges that increase significantly the complexity of the formulations, since, according to the economic theory embraced for this investigation, the impact—or cost—on society is a non-linear function of time. As part of an ongoing research project, this paper discusses the unique characteristics of the logistics problem faced during the distribution of critical goods in the aftermath of extreme events and presents formulations for vehicle routing that pursue the minimization of both operational and social costs. These formulations provide a new and integrated perspective to humanitarian logistics because they recognize explicitly the significant impacts that the logistical decisions may have on the victims’ welfare and vice versa.

The remainder of this article is organized as follows. Section 2 discusses the literature on humanitarian logistics. Section 3 provides an overview of some of the issues associated to the valuation of suffering. This section also discusses some general characteristics of the social cost function and its implications for humanitarian logistics modeling. Section 4 proposes formulations for the vehicle routing problem that
considering the deprivation costs associated with human suffering. The paper ends with a summary of the key findings in Section 5.

2. LITERATURE REVIEW

A basic distinction between commercial and humanitarian logistics is related to the different objectives that they pursue. The primary objective of commercial supply chains is to minimize the total logistics costs associated with the firm’s operations. Although operational costs are also relevant in disaster response, they are of lesser importance for humanitarian supply chains primarily concerned with expediting the delivery of critical goods to the affected areas to minimize negative impacts on the victims. This is a crucial distinction because, in the immediate aftermath of extreme events, disaster response organizations are usually unable to satisfy all needs and have to decide how best to use their limited resources. In doing so, they attempt to maximize the benefits from the aid delivered by minimizing the human suffering associated with the lack of access to a given good or service, i.e., deprivation cost. In contrast, commercial supply chains rarely, if ever, consider the social externalities from their delivery actions as a parameter for decision making under normal business conditions.

Since commercial and humanitarian supply chains pursue significantly different objectives, the models used in each setting could also be expected to differ in order to allow each supply chain to fulfill its particular purpose. However, a review of the literature available indicates that most formulations for humanitarian logistics are extensions of methods for commercial logistics that do not fully account for the particular challenges of humanitarian operations. As summarized in Table 1, some of these models focus on operational costs only, e.g. (3) and (4). Minimizing operational costs is certainly an important consideration in humanitarian logistics since a cost-effective response increases the share that the impacted areas receive from the financial resources marked up for the assistance to the victims. This is particularly important for non-governmental organizations relying on donations to support their operations, as donors understandably expect their contributions to provide maximum benefits to the victims. However, a low-cost response may also mean low levels of service to the affected population, with potentially detrimental effects that may be life-threatening in extreme cases.

The need to develop models reflecting the unique characteristics of humanitarian logistics has been acknowledged recently by a number of researchers that have devised techniques that somehow prioritize concerns for the affected individuals over the operational costs. For the most part, these methods propose a number of proxy measures to approximate the cost that particular distribution strategies impose on society. Some models focus exclusively on the minimization of unsatisfied demands, e.g. (5). Other common technique is to use priority or penalty factors to encourage the satisfaction of the most urgent needs (e.g. (6), (7), (8) and (9). Other formulations add equity constraints to ensure a minimum frequency of deliveries or a “fair” distribution of resources. These objectives are pursued by introducing constraints requiring a minimum number of deliveries to the impacted areas, e.g. (10), or by limiting the headway time between successive deliveries, e.g. (11). In addition to imposing levels of service that may be unfeasible given the usual discrepancies between supply and demand in the aftermath of extreme events, these models generally do not account for the operational costs incurred in responding to the disaster.

The use of proxy measures to approximate deprivation costs has other important limitations. For example, delivering as much assistance as possible may be a reasonable objective during the immediate response to a disaster, but financial considerations become increasingly important as the system stabilizes and operational costs overtake other concerns during decision making. Therefore, methods focusing solely on this objective are ineffective in identifying the resource allocation providing the highest benefit to the system. On the other hand, methods aiming at minimizing unmet demands do not distinguish a particular request from another, and thus are unable to account for the fact that the urgency with which supplies are
needed may vary among affected areas. Although models introducing equity constraints, priority or weight factors to encourage deliveries to certain areas are expected to provide more uniform levels of service, they require subjective considerations from the modeler regarding the definition of the equity, prioritization or weighting schemes. Therefore, their distribution solutions are highly sensitive to the specific parameter values used during the optimization.

Table 1: Summary of Humanitarian Logistic Formulations

<table>
<thead>
<tr>
<th>Formulation</th>
<th>Commercial Logistics</th>
<th>Unique to Humanitarian Logistics</th>
<th>Other objectives and/or constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimization of Operational Costs</td>
<td>Uncertain Demands</td>
<td>Variable Goods / Service Priority</td>
</tr>
<tr>
<td>Haghani and Oh (1996)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Barbarosoglu and Arda (2004)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>De Angelis et al. (2007)</td>
<td></td>
<td>(*)</td>
<td>(*)</td>
</tr>
<tr>
<td>Yunjun et al. (2007)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Chang et al. (2007)</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td>Balcik et al. (2008)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Rawls and Turnquist (2010)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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</tbody>
</table>

Note: (*) Propose solving deterministic versions of the problem as new information becomes available.

The few methods considering both operational costs and some proxy measure for the social implications of the delivery strategy, e.g. (12), (13) and (14), have difficulties in objectively reconciling these two
objectives. So far, this has been a controversial issue because of the challenges in evaluating the welfare of the affected individuals against operational costs. Unable to express operational and social costs in a common scale, these methods rely excessively on subjective parameters, thus biasing the optimization process towards the particular opinions of the decision makers. Therefore, one of the opportunities for improving humanitarian logistics modeling is to develop methods to properly capture this dichotomy in objectives.

3. SOCIAL COSTS: A NEW PARADIGM FOR HUMANITARIAN LOGISTICS
The unique characteristics of humanitarian supply chains and the shortcomings of current models suggest the need for new paradigms for humanitarian logistics. Rather than defining operational success in terms of travel time, total cost, or even cargo delivered, the ultimate performance measure for humanitarian organizations is how much human suffering they alleviate or prevent with their resources. No other objective captures better the fundamental mission of humanitarian logistics. Today, many organizations active in disaster response state the minimization of suffering as their primary goal, but, to the authors’ knowledge, none of them explicitly considers victims’ wellbeing as part of their supply chain models.

If the minimization of suffering is a logical and sensible approach to humanitarian logistics, then it should be quantified and incorporated to the models used in the planning and execution of response operations, a concept first proposed by (2). These authors suggest estimating social costs as a function of, e.g., commodity type and deprivation times, to capture the loss in welfare experienced by the victims of extreme events. They define deprivation cost as the economic value of the human suffering associated with not having access to a good or service. According to this approach, the optimal solution to a humanitarian distribution problem would be the strategy minimizing the sum of all costs incurred in responding to the emergency, both operational and social.

Using human suffering as a decision criterion leads to a series of philosophical, ethical and technical dilemmas: (a) ideally no one should be purposely left in peril (which may be an outcome); (b) it is certainly controversial to assign an economic value to suffering and the loss of lives; and (c) there is no easy way to adjudicate a cost to human suffering because of the lack of a proper market for it. (2) recognize these difficulties, but argue that humanitarian logistics is not the only field struggling with these complex issues. They show that the valuation of human suffering is not a new concept but rather a topic extensively studied by economists, and with important practical applications. Among the situations in which the value of human suffering is a primary factor for decisions making, these authors highlight the rationing schemes for organ transplantation (18), the development of health policy approaches (19), the financial evaluation of an individual’s life in the life insurance industry (20), and the triage systems (21).

(2) also discuss the economic theory behind their proposed use of social costs for humanitarian logistics. They argue that the Principle of Utility first discussed by philosophers Jeremy Bentham in 1789 and John Stuart Mill in 1863 ((22) and (23)) may offer the most convincing moral justification for the aggregation of deprivation costs when making decisions about how to “best” distribute critical supplies in the aftermath of large disasters. This principle states that “…actions should be judged by their consequences and that actions are right or good insofar as they produce the greatest net benefit among all those affected…” (21). In essence, because the demand for critical goods in the aftermath of large disasters usually exceed supply, some demand may not be satisfied in a timely manner or in the worst cases may not be satisfied at all; but these actions from the responder may be justified if they produce the greatest aggregate benefit to victims at the system level (2). Notwithstanding the extensive literature on the subject, the fundamental question of how to develop “fair” metrics for human suffering still remains a controversial issue. This paper simply assumes that such metric exists and is provided as an input to the models.
In spite of philosophical concerns and the difficulties associated to the in estimation of the social cost functions –two important topics beyond the scope of this paper– valuating suffering may be the most appropriate way to ensure fairness and equity in humanitarian response operations. In addition to representing better the objectives of humanitarian supply chains, this approach also offers several practical advantages over current methods. For example, the delivery constraints commonly used in humanitarian logistics, e.g., specifying that a delivery must be made to communities that have not received goods for a certain period of time, try to impose a level of service that may not be consistent with the limited resources available. Explicitly modeling the social costs associated to logistical decisions eliminates the need to define these subjective levels of service. Another advantage is that considering social costs allows for the unbiased tradeoff analysis between the social benefits associated with a delivery strategy, and the operational costs incurred in executing it. Moreover, incorporating social costs in the objective function allows also for the unbiased evaluation of opportunity costs, i.e., the costs incurred by all other nodes in which no deliveries took place. Although opportunity costs are a critical issue for responders to extreme events because allocating scarce resources to a particular area imposes deprivation costs on all other victims in need for the same goods, these costs are not considered in current models for humanitarian logistics.

Although social costs are subjective in nature due to individual differences in willingness to pay, (2) assert that certain characteristics can be assumed universal for the purposes of humanitarian logistics. For example, these cost functions should be commodity-dependent to represent the varying degrees of desirability and importance for human welfare among the requested goods. Social costs associated to humanitarian operations can also be safely assumed to be time-dependent, that is, the potential harm on victims increase as time goes by without their needs being fulfilled. These costs are also likely to be non-linear since the physiological damages faced by the victims increase rapidly in severity the longer the deprivation times. Even though people can sustain some physical and sociological distress for extended periods, the ability to cope diminishes with deprivation time. Nonlinearity of social costs can also be expected with respect to commodity types: the more critical the good, the steepest the cost function.

To help illustrate the concept, assume that the suffering produced by the lack of a particular good \( c \) is a function of the deprivation time, \( \tau \), and that its economic cost could be represented by a monotonic function \( \Gamma_{c}(\tau) \). Figure 1 illustrates the temporal impact of delivery actions on social costs. As shown by the discontinuous curve \( SC(t) = \Gamma_{c}(\tau) \), deprivation costs first increase with deprivation time, and then drop to zero when a delivery large enough to fulfill the needs of all individuals is received at time \( t_{1} \). However, by that time victims had already experienced a deprivation cost \( \gamma_{1} \) that cannot be removed by the commodities just received. Time \( t_{2} \) presents the case when the supplies delivered are less than the required amount to serve all the victims in the area. In this case, deprivation costs resets to zero for those who received the commodity and continue increasing for the victims who did not receive any goods, and thus the total social costs at the node decreases only by \( \gamma_{2A} - \gamma_{2B} \) in Figure 1. Such a partial delivery is assumed to satisfy the needs of one segment of the population only, which would then have a different willingness to pay for that commodity than those who were underserved. Consequently, decisions to make additional deliveries to that node should consider the needs of these two segments independently. For modeling purposes, this is equivalent to having two distinct nodes, i.e. the network has to be expanded to capture the differences in deprivation times. Since in the proposed approach for humanitarian logistics each victim in the system is a potential source of social costs, it is theoretically possible that many small deliveries may require the modeler to split a demand node in as many subnodes as individuals in it, likely to eventually lead to an intractable problem. An option to contain the sprawl of the resulting time expanded network is to define minimum delivery quantities, e.g. a fraction of the vehicle capacity,
which would cap the maximum number of subnodes that could be generated by the split deliveries. The models presented in the next section assume uniform delivery quantities for all the nodes and subnodes in the network.

\[ \gamma_1, \gamma_2, \gamma_3, \gamma_4, \ldots \]

\[ \text{Deprivation Costs} \]

\[ TSC(t) \]

\[ SC(t) \]

\[ t_1, t_2, t_3 \]

\[ \text{Time} \]

\[ \text{Figure 1: Impacts of delivery actions on social costs} \]

The incorporation of social costs to the models for humanitarian logistics presents additional technical challenges. For example, consider the cumulative social costs for the problem in Figure 1, represented by the curve \( TSC(t) \) on that plot. As shown, \( TSC(t) \) is non-convex, even though it is obtained from the summation of social costs determined with the nonlinear but convex \( \Gamma_c(\tau) \). Nonlinear and nonconvex problems are difficult to solve since traditional solution methods cannot guarantee finding optimal solutions. Furthermore, as previously discussed, deprivation times are critical parameters for the social costs associated to a delivery strategy. Deprivation times, in turn, are affected by both the timing and the size of the deliveries, i.e., large amounts of commodities could be stored in the affected area to sustain victims for extended periods of time. Tracking inventory levels, consumption, deliveries and vehicle trips in order to price correctly the distribution strategies require a large number of binary variables that may become computationally prohibitive for large problems. Hence, it is imperative to develop efficient methods to reduce computational requirements. For example, the total social costs up to time \( t_3 \) for the case in Figure 1 can determined by evaluating the cumulative cost function at that time epoch, i.e. \( TSC(t_3) \). However, keeping track of the evolution of the social costs at every node and time period require a large number of variables. It can be observed in Figure 1 that the cumulative social costs at time \( t_3 \) also equal \( \gamma_1 + \gamma_2 + \gamma_3 - \gamma_2 \), i.e., it suffices to calculate social costs at the discrete time epochs at which deliveries were made to the node and at the end of the planning horizon. This insight enables to simplify computations by only scanning the epochs for the alternative delivery actions considered. The following section presents formulations for the inventory routing problem incorporating social cost functions, along with other humanitarian logistics issues discussed in this paper.

4. THE VEHICLE ROUTING PROBLEM FOR HUMANITARIAN LOGISTICS
This section presents a vehicle routing and scheduling formulation for the distribution of critical goods (e.g., water, food, blood, etc.) in the aftermath of large disasters. Delivering supplies were most in need requires the consideration of existing inventories, if any, in the regional warehouses as well as in the local distribution points at the affected areas. Therefore, the problem targeted in this research resembles vendor managed systems, in which a centralized supplier (in this case, the responder to the disaster) is responsible for satisfying the demands from several customers (victims in the case of emergencies). This model captures characteristics of humanitarian logistics absent in other distribution problems.

4.1 Assumptions

It is assumed that the responder has good visibility over the entire system, which is defined as (1) a set of capacitated distribution centers with known location and initial inventory levels, (2) sets of heterogeneous vehicles for multiple transportation modes, with known capacities and initial locations, (3) transportation networks for each transportation mode, and (4) areas affected by the emergency. It is also assumed that, for each of the impacted sites, the responder has reliable estimates for the number of victims, the amount and type of goods needed and the resources initially available on the area, if any. To further simplify the formulation, some additional assumptions were made:

1. It is assumed that the time-dependent subjective value people place on critical supplies has been previously quantified and properly calibrated. Function $\Gamma_c(\Delta t)$ captures deprivation costs based on commodity type $c \in C$ and deprivation time $\Delta t = t - t'_{cit}$. $\Gamma_c(\Delta t)$ is further assumed convex and uniform for victims across all social groups (i.e., willingness to pay normalized to represent the average individual).

2. The responder knows the consumption rates $u_c$ for the set of goods $c \in C$ to be distributed, i.e., how many units of a particular commodity are required to sustain a person for one unit of time. This rate is assumed uniform for all people, regardless of age or other physiological characteristics. Even though consumption occurs at a discrete rate, in this research these rates are assumed continuous over time.

3. In this formulation, there are no request orders from demand nodes. The planner estimates demand based on the number of victims in a node and the consumption rates for each commodity. Backordering for unmet demands is not allowed (e.g., it is not reasonable to assume that a person who has not eaten in three days will consume three times the daily portions when supplies finally arrive.) Furthermore, demands are not removed from the system once deliveries are received; requests for additional deliveries regenerate automatically based on the number of victims at the demand nodes.

4. Travel times per mode, which include the loading and unloading of vehicles, are known for all travel arcs in the transportation networks and correspond to the shortest paths between nodes. Travel times are also assumed uniform for all vehicles of the same transportation mode. Although this model considers only travel costs, other operational metrics such as holding costs may be added with minor changes to the formulation.

5. Vehicles are assumed to operate continuously, with no limits on trip length or duration. Realistically, individual drivers cannot work safely beyond certain number of hours per shift. However, this model assumes multiple drivers per vehicle, a frequent practice in humanitarian operations.

6. In a large emergency, it may not be possible to reach all affected areas with out-and-back vehicle tours from distant distribution centers. In this formulation, vehicles are not assigned to particular
warehouses or even required to return to a depot at the end of the optimization. Vehicles are also allowed to wait in nodes and to pick-up commodities from any distribution center in the system. Moreover, demand nodes may also be used as transshipment points where commodities could be transferred among vehicles or transportation modes. A vehicle tour is defined as the sequence of visits to demand nodes between two consecutive stops at distribution centers to reload. Vehicles could complete multiple tours for relatively long planning periods.

7. Multiple and split deliveries are allowed in the model, though a minimum delivery quantity \( q_c \) is enforced for each commodity \( c \in C \). This minimum quantity is assumed large enough to serve all victims in a node for at least one period of time (i.e., no demand split: serve all or none of the victims in the node), and small enough not to exceed vehicle capacity. Nodes with large number of victims are split in equally populated replicate nodes such that the demand for commodities at each node is at most equal to the minimum delivery quantity. Delivery quantities in excess of the defined minimum are allowed by connecting all replicate nodes by no cost arcs (representing instantaneous travel time).

4.2 Notation

Before the mathematical model is presented, the notation used in the formulation is introduced below.

Input parameters:

\[
\begin{align*}
T & : \text{ length of the planning horizon } \\
N & : \text{ set of all nodes in the network } \\
DN & : \text{ set of demand and transshipment nodes, } DN \subset N \\
SN & : \text{ set of supply nodes, e.g. warehouses, depots and distribution centers, } SN \subset N \\
A_m & : \text{ set of arcs linking nodes in the transportation network for mode } m \in M \\
C & : \text{ set of commodities to be distributed } \\
M & : \text{ set of transportation modes } \\
K_m & : \text{ set of vehicle types for transportation mode } m \in M \\
u_c & : \text{ consumption rate, i.e., standard amount of commodity type } c \in C \text{ required to sustain an affected person for one unit of time} \\
\tau_{mij} & : \text{ travel time required to traverse arc } (i, j) \in A_m \text{ by transportation mode } m \in M \\
t_c^{\text{max}} & : \text{ maximum deprivation time a person could survive without commodity } c \in C \\
P_{it} & : \text{ population (number of victims) in node } i \in DN \text{ at time } t \\
q_c & : \text{ minimum delivery quantity for commodity } c \in C \\
Q_{km} & : \text{ load capacity for vehicle type } k \in K_m, \text{ mode } m \in M \\
c_{km} & : \text{ travel cost per unit of time for vehicle type } k \in K_m, \text{ mode } m \in M \\
w_c & : \text{ unit weight of commodity } c \in C \\
d_{ct} & : \text{ amount of commodity } c \in C \text{ added from external sources (e.g., donations) to node } i \in N \text{ at time } t \\
B & : \text{ a big number} \\
\delta_{km} & : \text{ 1 if vehicle } k \in K_m \text{ of mode } m \in M \text{ is located at node } i \in N \text{ at time } t = 0 \\
0 & \text{ otherwise}
\end{align*}
\]
\[
\begin{align*}
\ell_{i,t}^{km} & = \begin{cases} 
1 & \text{if vehicle } k \in K_m \text{ of mode } m \in M \text{ is added to the fleet at node } i \in N \text{ at time } t \\
0 & \text{otherwise}
\end{cases} \\
\text{Decision variables:} \\
x_{ijt}^{c,m} & : \text{amount of commodity type } c \in C \text{ shipped from node } i \text{ to node } j \text{ at time } t, \text{ using vehicle } k \in K_m \text{ of transportation mode } m \in M \\
I_{cit} & : \text{inventory level, i.e., amount of commodity type } c \text{ carried over from time } t \text{ to time } t+1 \text{ at node } i \\
t_{cit} & : \text{last time at which people in node } i \in DN \text{ consumed commodity } c \in C \text{ previous to current time } t. \\
\alpha_{cit} & \left\{ \begin{array}{ll} 1 & \text{if } I_{cit} \geq u_c P_{it} \text{ (node } i \text{ has enough inventory of commodity } c \text{ for at least one period of time)} \\
0 & \text{otherwise} \quad \text{(a shortage for commodity } c \in C) \\
\phi_{cit} & \left\{ \begin{array}{ll} 1 & \text{if commodity } c \text{ arrives to node } i \text{ at time } t \text{ (by any transportation mode)} \\
0 & \text{otherwise}
\end{array} \right.
\end{array} \right.
\end{align*}
\]

4.3 The Vehicle Routing Problem
The objective of the vehicle routing problem (VRP) is to determine the most efficient allocation and distribution of critical goods to the affected areas, considering both operational and deprivation costs. The problem may be formulated as follows:

\[
\begin{align*}
\text{Minimize} \quad & \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A_n} \sum_{t=1}^{T} c_{ijt}^{m} x_{ijt}^{c,m} + \sum_{c \in C} \sum_{i \in DN} \sum_{t=1}^{T} \phi_{cit} (1-\alpha_{cit,t-1}) \cdot \Gamma(t-t_{cit,t}) \\
& \quad + \sum_{c \in C} \sum_{i \in DN} P_{it} (1-\alpha_{cit}) \cdot \Gamma(T-t_{cit}) \\
\text{s.t.} \\
I_{cit,t+1} + a_{cit}^S = I_{cit} + \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A_n} x_{ijt}^{c,m} \quad \forall i \in SN, c \in C, t \in 1..T \quad (2) \\
I_{cit,t-1} + a_{cit}^S + \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A_n} x_{ijt}^{c,m} - \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A_n} x_{ijt}^{c,m} - u_c P_{ijt} \alpha_{ci,t-1} = I_{cit} + \sum_{m \in M} \sum_{k \in K} \sum_{(i,j) \in A_n} x_{ijt}^{c,m} \quad \forall i \in DN, c \in C, t \in 1..T \quad (3) \\
\phi_{ij,t-1} q_c \leq x_{ijt}^{c,m} \quad \forall c \in C, (i,j) \in A_m \quad \forall j \in DN, m \in M, k \in K, \quad t \in 1..T \quad (4) \\
I_{cit} - u_c \left( P_{it} - \frac{i}{2} \right) \leq \alpha_{cit} B \quad \forall i \in DN, c \in C, t \in 1..T \quad (5)
\end{align*}
\]
The objective function to be minimized in equation (1) captures the total costs incurred during a planning period of length $T$. The first term in this equation represents the total travel and handling costs for delivering commodities to the affected areas. The other two terms in the objective function correspond to the social component of the formulation: the deprivation costs that were alleviated by delivering commodities to the nodes (e.g. $SC(t_i)$ and $SC(t_j)$ in Figure 1), and the deprivation costs being experienced by victims in all nodes at the end of the planning period (e.g. $SC(t_j)$ in Figure 1). This function meets the intended objective of combining both operational and social costs in a single function, and collapses to a minimum travel cost problem during the recovery stage, when it is assumed that the responder is able to meet all demands on time (thus avoiding social costs).

Constraints (2) balance commodity flows at the supply nodes. Likewise, constraints (3) enforce the conservation of commodity flows at supply nodes, allowing for commodity transfers among vehicles and transportation modes. Parameter $a_{cit}$ in these two sets of constraints represents the expected arrival of commodities from sources exogenous to the system, which could be in the form of donation pledges or in-transit shipments from external sources. The term $u_c P_{it} \alpha_{cit}$ in constraints (3) defines the total consumption of commodities at nodes per unit of time, and restricts consumption to instances where the commodity is available, i.e., when inventory at a node exceeds at least the requirements for one period of time. This expression explicitly defines demand based on the number of victims in a node, $P_{it}$, and the commodity consumption rate per person, $u_c$. As expected, commodities may be required as long as there
are victims at a node, even if goods are delivered (in typical VRP applications, the demands are discrete occurrences that are removed from the problem once satisfied).

Constraints (5) restrict the amount of commodity flowing between nodes at any time to be at least as large as the minimum delivery quantity defined for the particular commodity type. Constraints (5) and (6) combine to define the binary variable $\alpha_{cit}$ to indicate the existence of inventory ($\alpha_{cit} = 1$) or shortage ($\alpha_{cit} = 0$) at the demand nodes. It is assumed that a shortage of magnitude $u_c P_t$ occurs when there is no inventory at the node or when the supplies available are insufficient to meet needs of all its population for at least one period of time. This conservative stance penalizes a distribution plan for incurring shortages when the inventory level at a demand node falls below the minimum required to sustain all victims, since some people there will be underserved as inventory depletes completely sometime during period $[t, t+1]$.

Constraints (7) and (8) combine to define the binary variables $\phi_{cit}$ that indicate the arrival of commodities to demand nodes. Social costs increase continuously with deprivation times, but are only charged in the objective function when $\phi_{cit}$ signals that a delivery was received, which in turns reset the deprivation costs at that node to zero, i.e., no hysteresis. Constraints (9) track $t'_{cit}$, the variables indicating the last time prior to current time $t$ at which the population at node $i$ consumed commodity $c$. Constraints (10) prohibits deliveries to nodes that have been short of a critical good $c$ for a period of time exceeding $t_{max}$, the maximum time a person could survive without that commodity. It is assumed that people would die after this threshold value, and therefore commodities are no longer needed at that node. It is assumed that such occurrence is penalized severely by the social cost function.

Inequalities (11) and (12) allow for the construction of vehicle routes by requiring that every vehicle enters a node or leaves it, respectively, at most once at any point in time. Constraints (13) enforce the conservation of vehicle flows by allowing a vehicle to departure from a node at time $t$ only if its cumulative number of entries up to that time exceeds the number of departures from the node. The initial location and the addition of a vehicle to fleet at a node are also considered as entries to the node in these constraints.

Constraints (14) link commodity and vehicle flows by allowing commodities to traverse arc $(i,j)$ in vehicle $k$ only if that vehicle is traveling along the arc simultaneously, without exceeding its load capacity. Finally, constraints (15) define the variables for the model.

The model above provides an explicit representation of the vehicle routing problem, and shows explicitly the increased difficulties faced by responders to humanitarian emergencies. However, the resulting formulation is a complex non-linear combinatorial problem that may not be solved by exact methods. Although it incorporates both operational and social costs, the objective function (1) is non-linear and non-convex; further complicated by the fact that the objective function may experience sharp and discontinuous changes in value whenever deliveries are made. Moreover, the model requires two large sets of binary variables ($\alpha$ and $\phi$) and the non-linear constraint set (9) to capture key determinants for the social costs (i.e., deprivation times, the existence of inventory at demand nodes and the arrival of deliveries to impacted areas). A third set of binary variables $\theta$ is also necessary to track the movement of every vehicle in the system. A relatively minor issue with this model is that the two “Big M” constraint sets (5) and (7) require the careful definition of tight bounds to reduce the solution space for the MIP. These characteristics increase significantly the complexity of the already difficult vehicle routing problem –even traditional VRP applications with much simpler formulations are notoriously NP-hard--, and thus require the development of heuristics for problems of realistic size. The next section presents an alternative network formulation and provides some insight into potential solution methods.
4.4 Multicommodity linear network formulation

Similar to the model presented in the previous section, many formulations for inventory routing problems track vehicles individually on a route basis by representing them as decision variables. The advantage of those models is that they find the optimal tours and delivery schedules in a single optimization step. However, these combinatorial problems, even with deterministic or linear costs, are computationally prohibitive for the kind of multiperiod applications discussed in this research, characterized by both pick-ups and split deliveries, multiple depots, products and transportation modes.

It may be possible to reduce the computational burden while still accounting for these characteristics by regarding routing decisions as a secondary problem. In this approach, vehicles are modeled as commodities flowing in a minimum cost network rather than as binary variables. Although it requires additional post-processing to extract the vehicle routes from the optimal commodity flows, treating vehicles as commodities allows for the exploitation of special network structures that may be solved more efficiently than the single-step vehicle routing and scheduling problem. The formulation in the previous section may be further simplified by converting the problem to a linear program. Let:

\[ \gamma_{cd} = \Gamma_c(d) - \Gamma_c(d - 1), \] i.e., the marginal deprivation cost associated with not having commodity \( c \) during period \( [d-1, d) \), \( d \in 1..T^\max \)

- \( a_{lim}^m \): number of vehicles type \( k \), mode \( m \), added or removed from the fleet at node \( i \) at time \( t \)
- \( x_{ijt}^m \): amount of commodity type \( c \) shipped by mode \( m \) from node \( i \) to node \( j \) at time \( t \)
- \( y_{ijt}^m \): number of vehicles type \( k \), mode \( m \), routed from node \( i \) to node \( j \) at time \( t \)
- \( V_{t}^{km} \): number of vehicles type \( k \), mode \( m \), stationed at node \( i \) at time \( t \)
- \( \delta_{it}^{S_{c}} \): \[ \begin{cases} 1 & \text{if node } i \text{ has been without commodity } c \in C \text{ for } d \in 1..T^\max \text{ units of time at time } t \\ 0 & \text{otherwise (a shortage for commodity } c) \end{cases} \]

Using these parameters and variables in conjunction with those defined in Section 4.2, the alternate multicommodity network formulation for the distribution of critical goods in the aftermath of extreme events can be expressed as shown below.

\[
\text{Minimize} \quad \sum_{m} \sum_{k} \sum_{(i,j) \in A_e} \sum_{t=1}^{T} c_{ijt}^m \gamma_{ijt}^m + \sum_{i \in SN} \sum_{c \in C} \sum_{d=1}^{T} \sum_{t=1}^{T} \delta_{it}^{S_{c}} P_{it} \left( \gamma_{cd} - \gamma_{c,d-1} \right) 
\]

\[
\text{s.t.}
\]

Commodity flows:

\[
I_{cit} = I_{cit} + \sum_{m \in M} \sum_{(i,j) \in A_e} x_{ijt}^m \quad \forall i \in SN, c \in C, t \in 1..T
\]

\[
I_{cit} = I_{cit} + \sum_{m \in M} \sum_{(i,j) \in A_e} x_{ijt}^m - \mu_c P_{it-1} \alpha_{ct} = I_{cit} + \sum_{m \in M} \sum_{(i,j) \in A_e} x_{ijt}^m 
\]

\[ \forall i \in DN, c \in C, t \in 1..T \]
\[ I_{ct} - u_c(P_c - \frac{1}{2}) \leq \alpha_{ct} \sum_{i=1}^{\text{max}} u_i P_i \quad \forall i \in DN, c \in C, t \in 1..T \] (19)

\[ (u_c P_i) \alpha_{ct} \leq I_{ct} \quad \forall i \in DN, c \in C, t \in 1..T \] (20)

\[ \delta_{ct,t+1}^d = 1 \quad \forall i \in DN, c \in C \] (21)

\[ \delta_{ct}^d \geq \delta_{ct,t-1}^d - \alpha_{ct,t-1} \quad \forall i \in DN, c \in C, t \in 1..T, d \in 1..t_c^{\max} \] (22)

\[ \delta_{ct,t}^{d0} \geq \alpha_{ct,t-1} \quad \forall i \in DN, c \in C, t \in 1..T \] (23)

Vehicle flows:
\[ V_{it}^{lm} + a_{nit}^V + \sum_{(j,i) \in A_n} Y_{jm}^{lm} = V_{it}^{lm} + \sum_{(i,j) \in A_n} Y_{ji}^{lm} \quad \forall i \in N, k \in K_m, m \in M, t \in 1..T \] (24)

Constraint linking commodity and vehicle flows:
\[ \sum_{c} w_c x_{ij}^{cm} - \sum_{t} Q_x^{lm} Y_{it}^{lm} \leq 0 \quad \forall (i, j) \in A_n, c \in C, k \in K_m, m \in M, t \in 1..T \] (25)

Definition of commodity flow variables:
\[ x_{ij}^{cm} \geq 0, \ I_{ct} \geq 0, \ \alpha_{ct} = \text{binary}, \ \delta_{ct}^d = \text{binary} \] (26)

Definition of vehicle flow variables:
\[ Y_{it}^{lm} \geq 0 \text{ and integer}, \ V_{it}^{lm} \geq 0 \text{ and integer} \] (27)

The objective (16) is a linear function minimizing travel and deprivation costs. Contrary to the objective (1) for the VRSP, deprivation costs are discretized by time unit and added to the objective function as they are realized by the victims, rather than as a lump amount added only at the time of delivery. This transforms the objective into a continuous function, although the discontinuities in (1) are replaced by still non-differentiable sharp edges or kinks. Constraints (17) and (18) ensure conservation of commodity flows at supply and demand nodes, respectively. In these constraints, commodity flows \( x_{ij}^{cm} \) are independent from vehicle flows. Constraints (19) and (20) are the same as (5) and (6) defining the binary variables \( \alpha_{ct} \). A tight bound on the large number \( B \) required in (5) is explicitly modeled in (19) as \( \sum u_c P_i \).

Constraint set (21) initializes the binary variables \( \delta_{ct}^d \) tracking social costs at each node, while inequalities (22) ensure that those costs are increased systematically until deliveries are received. Deliveries force \( \alpha_{ct} = 1 \) at that node, which in turn resets its deprivation cost function, i.e. \( \delta_{ct,t+1}^{d0} = 1 \) according to inequalities (23). If deprivation time exceeds \( t_c^{\max} \) at any node, the minimization of costs would prevent \( \delta_{ct}^d \) from resetting at that node, which then makes unnecessary to model this restriction explicitly as in (10). Therefore, this formulation relies on the interaction between variable sets \( \alpha_{ct} \) and \( \delta_{ct}^d \) to implicitly track deprivation times and deliveries, which were previously tracked explicitly by \( t_{ct} \).
and $\phi_{ci}$ in the VRSP model. Variables $\delta_{ci}^d$ also enable the linearization of the problem and the smoothening of the objective function. These advantages, however, come at the expense of a large number of binary variables.

The set of constraint (24) in the model correspond to the vehicle flows. These constraints enforce the conservation of vehicle flows at all times, considering both the possibility of a vehicle waiting in an node and the arrival of any additional vehicles, $a_{ci}^S$, to the system. Constraints (25) link commodity and vehicle flows by conditioning the flow of commodities between nodes to the existence of enough vehicle capacity on that arc. This constraint prevents the indiscriminate distribution of supplies in the network and foster feasible vehicle tours and delivery schedules. Constraints (26) and (27) define the variables for the model.

A limitation of modeling the inventory routing problem for humanitarian logistics as a multicommodity network problem is that the solution to this formulation provides only the aggregate optimal vehicle and commodity flows for the system. Thus, additional post-processing is needed to construct the actual vehicle tours and delivery schedules. However, the complexity of this additional step is much reduced from the NP-hard combinatorial problem in the VRSP formulation, as optimal flows are no longer decision variables but (fixed) input parameters to the second stage problem. Several heuristics have been discussed in the literature for this problem, finding solutions in polynomial time, e.g. (3) and (8).

Even with its network structure, the model above is expected to find optimal solutions only to relatively small problems. It will be necessary to develop heuristics to approximate the optimal solution for scenarios of realistic size. A particular feature of the multicommodity network formulation is that, by relaxing constraints (25), the model decomposes in two independent mixed integer problems: one for the commodity flows and other for the vehicle flows. This special characteristic may enable the use of powerful heuristics to solve large instances of this inventory routing problem. For example, constraints (25) could be relaxed using Lagrangian multipliers, a method that has been effective in finding close to optimal solutions to other complex problems, e.g. urban traffic networks.

Another limitation of the network model is that it does not consider the dynamic nature of the system. There is no uncertainty in this formulation, as it assumes all demands, resources and travel times as deterministic parameters known at the time of the optimization. However, this is not the case during the immediate response to large emergencies. One way to deal with uncertainty is to run a new optimization when conditions change, but this approach may not be feasible for real-time situations. It is thus necessary to develop methods to efficiently incorporate new information as it becomes available. Some options currently being explored by the research team include metaheuristics and insertion methods.

6. CONCLUSIONS AND FUTURE RESEARCH
This paper discusses the need for analytical formulations specifically designed for humanitarian operations. Although commercial and humanitarian logistics pursue the same general objective of delivering goods to points of demand, they differ in key aspects. A fundamental difference is that commercial logistics aim to minimize the operational costs incurred by the business, while social costs are a more important concern for humanitarian logistics, at least during the immediate response to a disaster. A review of state of the art models in humanitarian logistics suggests that current formulations do not fully account for this and other unique challenges faced by responders delivering critical goods in the aftermath of extreme events, as they focus on performance measures better suited for commercial logistics. Moreover, the paper argues that the equity constraints and penalty factors commonly used in humanitarian logistics are arbitrary in nature and likely to lead to infeasible or suboptimal solutions that may increase human suffering at the impacted sites.
The paper argues that any distribution strategy will affect the victims’ welfare even if deprivation costs are not explicitly considered in the corresponding modeling methods. Such change in wellbeing can be quantified using economic valuation techniques and incorporated to the analytical formulations. In spite of the philosophical concerns and the difficulties associated to the in estimation of the social cost functions, this paper contends that valuating suffering may be the most appropriate way to ensure fairness and equity in humanitarian response operations because it bypasses the need to define subjective levels of service and allows for the tradeoff analysis between the social benefits attributed to a delivery strategy and the operational costs incurred in executing it.

Incorporating social costs to the decision making process significantly increases the complexity of the logistics models, since, according to the economic theory embraced for this investigation, social costs are a non-linear and function of time and commodity type. This paper presented two formulations for vehicle routing and scheduling that evidence the complex nature of these models. These formulations recognize explicitly the significant impacts that the logistical decisions may have on the victims’ welfare and define a new variation of the classical vehicle routing problem. The development of solution methods for problems of realistic size remains a topic for future research.

7. REFERENCES


