Objectives:

- To review fundamental relationships of traffic flow (speed, density, flow)

- To understand the basic models of traffic flow

- To understand principles of queuing theory.
Speed vs Flow:

\[ R = k_j \left( 1 - \frac{u}{u_f} \right) \]  

\[ q = u k \]  

\[ R = \left( \frac{q}{u} \right) \]

\[ \left( \frac{q}{u} \right) = k_j \left( 1 - \frac{u}{u_f} \right) \]

\[ q = k_j (u - k_j \left( u - \frac{u^2}{u_f} \right)) \]
5.3  \[ \text{Max } q \]

\[ q = 50k - 0.156k^2 \]

\[ \frac{dq}{dk} = 0 \]

\[ 50 - 0.312k = 0 \]

\[ k = \frac{\text{Capacity}}{4} \]
\[ K = \frac{50}{0.312} = 160 \, \text{veh/mi} \]

(i) \[ Q_{\text{cap}} = 50 (160) - 0.156 (160)^2 \]
\[ = 9006.4 \, \text{veh/hr} \]

(ii) \[ Q = u x k \]
\[ U_{\text{cap}} = \frac{Q_{\text{cap}}}{K_{\text{cap}}} = \frac{9006.4}{160} = 56.29 \, \text{mi/hr} \]

(iii) \[ Q = \frac{Q_{\text{cap}}}{y} = \frac{9006.4}{y} = 1001.6 \, \text{veh/hr}. \]
\[ 1001.6 = 50k - 0.156k^2 \]

\[ 0.156k^2 - 50k + 1001.6 = 0 \]

\[ k_1 = 21.5 \text{ veh/mi} \quad \rightarrow \quad (\text{uncongested}) \]

\[ k_2 = 299 \text{ veh/mi} \quad \rightarrow \quad (\text{congested}) \]

---

**Microscopic Details of Traffic Flow**

- **Uniform Speed**
- \(2 \text{ vehicles} \rightarrow 70 \text{ mi/hv} \)
- **Deterministic Model**
- \(1, 2, 3, 4, 5, \ldots, 3600\)
**Poisson Distribution**

\[ P(n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!} \]

**Model → Stochastic Models**

**Headway:**

\[ \frac{3600 \text{ veh/hv}}{t} = \lambda \]

\[ \lambda = \text{Average vehicle flow/Average rate of vehicles per unit time} \]

\[ t = \text{time duration of interest} \]

\[ e = \text{base of the natural logarithm} \ (e \approx 2.718) \]
Example 5.5

\[ 3 + 5 + 9 + \cdots + 101 = \frac{101}{15} = \frac{6.0667}{15 \times 60} = 0.112 \text{ veh/s} \]

\[ P(1) = \frac{(2t)^n e^{-2t}}{n!} \]

\[ P(n \geq 6) \text{ in 3 consecutive time intervals} \]

\[ P(0) = (0.112 \times 60)^0 e^{-0.112 \times 60} \]

\[ t = 60 \text{ seconds} \]

0!

\[ = e^{-6.732} = 0.0012 \]
\[ P(1) = (6.733)^1 \frac{e^{-6.733}}{1!} = 0.008 \]

\[ P(2) = (6.733)^2 \frac{e^{-6.733}}{2!} = 0.027 \]

\[ P(3) = 0.0606 \]

\[ P(4) = 0.102 \]

\[ P(5) = 0.137 \]

\[ 1 - P(n \leq 5) = \sum_{k=0}^{5} P(k) + P(6) + P(7) + P(8) \]
\[ 1 - 0.3358 = 0.6642 \]

\[ P(n \geq 6) = 0.6642 \]

\[ 12:15 \text{ to } 12:16 \Rightarrow 0.6642 \quad \Rightarrow \quad P(n \geq 6) \times P(n \geq 6) \times P(n \geq 6) \]

\[ 12:16 \text{ to } 12:17 \Rightarrow 0.6642 \quad \times \quad 0.6642 = (0.6642)^3 \]

\[ 12:17 \text{ to } 12:18 \Rightarrow 0.6642 \quad \times \quad 0.6642 = 0.293 \]

\[ \lambda = \frac{9}{3600} \text{ veh/5} \]

\[ \lambda = \frac{9}{3600} \text{ veh/5} \]
\[ P(n) = \left( \frac{9t}{3600} \right)^n \cdot e^{-\frac{9t}{3600}} \]

\[ n = 0 \Rightarrow Time \text{ HeadsUp}, h \]

\[ P(0) = P(h \geq t) = e^{-\frac{9t}{3600}} \]

\[ P(h \geq t) = e^{-\frac{9t}{3600}} \]
60% are 13 seconds or greater ...

\[ P(h \geq 13) = 0.6 \]

\[ P(h \geq t) = e^{-\frac{9t}{3600}} = 0.6 \quad t = 13 \]

\[ P(h \geq 13) = e^{-\frac{9 \times 13}{3600}} = 0.6 \]

\[ -\frac{9 \times 13}{3600} = \ln(0.6) \]

\[ \frac{9}{3600} = 141.9 \text{ veh/hr} \]

\[ 9 = 141.9 \text{ veh/hr} = \frac{141.9}{3600} = 0.039 \text{ veh/s} \]
\[ \lambda = 0.039 \times 30 = 1.182 \text{ veh/30 sec} \]

\[ P(n) = \frac{-2t \cdot e^{-2t}}{n!} \]

\[ 2t = 1.182 \]

\[ n = \lambda \]

\[ P(4) = (1.182)^4 \cdot e^{-1.182} \]

\[ \frac{4!}{4!} = 0.025 \]
Limitations of Poisson Model:

- Uncorrelated
- Light Traffic

Queueing Delays

Queue Formation, Queue Analysis

↓ Delays, Queue Length, Performance

Queueing Delays

→ Extreme Conditions → 90% of the Travel Time is Just Due to Delays
Queuing Models: 1. Estimate the performance of highways

Signalized Intersection:
- Delays (Time)
- Flow / Throughput
- Queue Length

Characteristics of a Queuing System:
- Arrival Process
- Departure Process
- No. of Servers
- Priority
- FIFO / LIFO
- Transportation