Outline

- Introduction
- Tractive Effort and Resistance
- Aerodynamic Resistance
- Rolling Resistance
- Grade Resistance
- Available Tractive Effort
  - Maximum Tractive Effort
  - Engine Generate Tractive Effort
- Vehicle Acceleration
- Fuel Efficiency
Outline

- Principles of Braking
  - Braking Forces
  - Braking Force Ratio and Efficiency
  - Antilock Braking Systems
  - Theoretical Stopping Distance
  - Practical Stopping Distance
  - Distance Traveled during Driver Perception/Reaction
Introduction

- Highway design and traffic analysis is based on performance of road vehicles.

- Examples:
  - Length of freeway
  - Highway grades
  - Stopping sight distance
  - Passing sight distance

- Vehicles performance is also utilize for design of traffic control devices and the timing of traffic signal control systems.
Introduction

Roads with vertical or horizontal curves have more strict speed limits

While straight roads have more freedom of movement therefore having higher speed limits
Introduction

- The objective of this chapter is to explain basic principles of vehicle performance.

- Why study vehicle performance?
  - Provides insight for highway design and traffic operations
  - Provides a basis to create more advanced vehicle technologies

- Why is this important?
  - Advances in vehicle technology requires updates based on highway design guidelines
  - Better understanding of vehicle performance
Tractive Effort and Resistance

- This are the two primary forces when determining the straight line performance of vehicles.
- Definitions:
  - Tractive Effort – force available at roadway surface which is able to create work on a vehicle. Units: Newtons (N).
  - Resistance – force that impedes vehicle motion. Units: Newtons (N).
Tractive Effort and Resistance

- There are 3 major sources of vehicle resistance:
  - Aerodynamic Resistance
  - Rolling Resistance
  - Gravitational Resistance

Figure 2.1 Forces acting on a road vehicle.
Tractive Effort and Resistance

**Figure 2.1** Forces acting on a road vehicle.

- $R_a =$ aerodynamic resistance in lb (N),
- $R_{rlf} =$ rolling resistance of the front tires in lb (N),
- $R_{rlr} =$ rolling resistance of the rear tires in lb (N),
- $F_f =$ available tractive effort of the front tires in lb (N),
- $F_r =$ available tractive effort of the rear tires in lb (N),
- $W =$ total vehicle weight in lb (N),
- $\Theta_g =$ angle of the grade in degrees,
- $m =$ vehicle mass in slugs (kg), and
- $a =$ acceleration in ft/s² (m/s²).
Tractive Effort and Resistance

- Adding forces at longitudinal axis:
  \[ F = F_f + F_r \quad \text{and} \quad R_r = R_{rlf} + R_{rlr} \]  
  \[ F = m \cdot a + R_a + R_{rl} + R_g \]  
  \[ R_g = \text{grade resistance} = W \cdot \sin \Theta_g \] (2.1) (2.2)
Future of Tractive Effort Technologies

http://www.youtube.com/watch?v=eT8nNFQ34WQ
Aerodynamic Resistance

- Can have significant impact on vehicle performance.
- Sources:
  - Turbulence around the body (85%)
  - Air friction (12%)
  - Air flow through components (3%)
- Basic Formula:

\[
R_a = \frac{\rho}{2} C_D A_f V^2 \quad (2.3)
\]
Aerodynamic Resistance

- Basic Formula Aerodynamic Resistance
  - $R_a =$ aerodynamic resistance in lb (N),
  - $\rho =$ air density in slugs/ft$^3$ (kg/m$^3$),
  - $C_D =$ coefficient of drag and is unit less,
  - $A_f =$ frontal area of the vehicle (projected area of the vehicle in the direction of travel) in ft$^2$ (m$^2$), and
  - $V =$ speed of the vehicle in ft/s (m/s).

- For more accurate results $V$ can be used as the speed of the vehicle relative to the prevailing wind speed.
### Aerodynamic Resistance

#### Chapter 2: Road Vehicle Performance

**Table 2.1** Typical Values of Air Density under Specified Atmospheric Conditions

<table>
<thead>
<tr>
<th>Altitude (ft)</th>
<th>Temperature (°F)</th>
<th>Temperature (°C)</th>
<th>Pressure (lb/in²)</th>
<th>Pressure (kPa)</th>
<th>Air density (slugs/ft³)</th>
<th>Air density (kg/m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>59.0</td>
<td>15.0</td>
<td>14.7</td>
<td>101.4</td>
<td>0.002378</td>
<td>1.2256</td>
</tr>
<tr>
<td>5,000</td>
<td>41.2</td>
<td>9.7</td>
<td>12.2</td>
<td>84.4</td>
<td>0.002045</td>
<td>1.0567</td>
</tr>
<tr>
<td>10,000</td>
<td>23.4</td>
<td>-4.5</td>
<td>10.1</td>
<td>70.1</td>
<td>0.001755</td>
<td>0.9996</td>
</tr>
</tbody>
</table>

**Table 2.2** Ranges of Drag Coefficients for Typical Road Vehicles

<table>
<thead>
<tr>
<th>Vehicle type</th>
<th>Drag coefficient ($C_D$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Automobile</td>
<td>0.25–0.55</td>
</tr>
<tr>
<td>Bus</td>
<td>0.5–0.7</td>
</tr>
<tr>
<td>Tractor-Trailer</td>
<td>0.6–1.3</td>
</tr>
<tr>
<td>Motorcycle</td>
<td>0.27–1.8</td>
</tr>
</tbody>
</table>
## Aerodynamic Resistance

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Drag coefficient ($C_d$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967 Chevrolet Corvette</td>
<td>0.50</td>
</tr>
<tr>
<td>1967 Volkswagen Beetle</td>
<td>0.46</td>
</tr>
<tr>
<td>1977 Triumph TR7</td>
<td>0.40</td>
</tr>
<tr>
<td>1977 Jaguar XJS</td>
<td>0.36</td>
</tr>
<tr>
<td>1987 Acura Integra</td>
<td>0.34</td>
</tr>
<tr>
<td>1987 Ford Taurus</td>
<td>0.32</td>
</tr>
<tr>
<td>1993 Acura Integra</td>
<td>0.32</td>
</tr>
<tr>
<td>1993 Ford Probe GT</td>
<td>0.31</td>
</tr>
<tr>
<td>1993 Ford Ranger (truck)</td>
<td>0.45</td>
</tr>
<tr>
<td>1997 Lexus LS400</td>
<td>0.29</td>
</tr>
<tr>
<td>1997 Infiniti Q45</td>
<td>0.29</td>
</tr>
<tr>
<td>2000 Honda Insight (hybrid)</td>
<td>0.25</td>
</tr>
<tr>
<td>2002 Acura NSX</td>
<td>0.30</td>
</tr>
<tr>
<td>2002 Lexus LS430</td>
<td>0.25</td>
</tr>
<tr>
<td>2003 Dodge Caravan (minivan)</td>
<td>0.35</td>
</tr>
<tr>
<td>2003 Ford Explorer (SUV)</td>
<td>0.41</td>
</tr>
<tr>
<td>2003 Dodge Ram (truck)</td>
<td>0.53</td>
</tr>
<tr>
<td>2004 Mercedes Benz C240</td>
<td>0.27</td>
</tr>
</tbody>
</table>
Aerodynamic Resistance

- **Drag Coefficient**
  - Recent vehicles have lower coefficients
  - Large personal vehicles have higher coefficients

Even minor factors as opening a window can have a significant effect on drag coefficients and therefore affecting total aerodynamic resistance.

Figure 2.2; page 11
Aerodynamic Resistance

- **Power required to overcome air resistance:**

  **US Customary**

  \[ \text{hp}_{Ra} = \frac{\rho C_D A_f V^3}{1100} \]

  **Metric**

  \[ P_{Ra} = \frac{\rho}{2} C_D A_f V^3 \]
Aerodynamic Resistance

Where:

- \( h_{Ra} \) = horsepower required to overcome aerodynamic resistance (1 horsepower equals 550 ft-lb/s),
- \( P_{Ra} \) = power required to overcome aerodynamic resistance in Nm/s (watts), and
- Other terms as defined previously.
Rolling Resistance

- Resistance generated from vehicle’s mechanical components and pneumatics as they generate friction by interacting with road surface.

- Sources:
  - **Tire deformation (90%)**
  - **Pavement penetration (4%)**
  - **Friction, other sources (6%)**

- Factors influencing sources:
  - **Tire inflation, temperature, speed**
Rolling Resistance

- It can be approximated as the product of friction terms and the weight of the vehicle, acting as a normal force on the road surface.

U.S. Customary

\[
f_{rl} = 0.01 \left(1 + \frac{V}{147}\right)
\]

Metric (2.5)

\[
f_{rl} = 0.01 \left(1 + \frac{V}{44.73}\right)
\]

Where:

- \(f_{rl}\) = coefficient of rolling resistance and has no units, and
- \(V\) = vehicle speed in ft/s (m/s).
Rolling Resistance

For most highway applications the angle of grades ($\Theta_g$) is very small the assumption of $\cos \Theta_g = 1$ can be used. Therefore:

$$R_{rl} = f_{rl} \times W \quad (2.6)$$

- **US Customary**

$$\text{hp } R_{rl} = \frac{f_{rl} \times \text{WV}}{550}$$

- **Metric**

$$P_{R_{rl}} = f_{rl} \times \text{WV} \quad (2.4)$$
Rolling Resistance

Where:

- $\text{hp}_{Rrl} = \text{horsepower required to overcome rolling resistance (1 horsepower equals 550 ft-lb/s),}$
- $\text{P}_{Rrl} = \text{power required to overcome rolling resistance in Nm/s (watts), and}$
- $W = \text{total vehicle weight in lb (N)}.$
Rolling Resistance

Example 2.1

A 2500-lb (11.1-kN) car is driven at sea level ($\rho = 0.002378$ slugs/ft$^3$ or 1.2256 kg/m$^3$) on a level paved surface. The car has $C_d = 0.38$ and 20 ft$^2$ (1.86 m$^2$) of frontal area and. It is known that a maximum speed, 50 hp (37.3 kW) is being expended to overcome rolling and aerodynamic resistance. Determine the car’s maximum speed.
Grade Resistance

- Gravitational force acting on a vehicle.
  \[ R_g = W \times \sin \Theta_g \]  
  (2.8)

- Highway grades are usually very small, so
  \( \sin \Theta_g = \tan \Theta_g \)
  \[ R_g = W \times \tan \Theta_g = W \times G \]  
  (2.9)

- Grades are expressed as percentages

\[ G = 0.05 \]
\[ \Theta_g = 2.86^\circ \]
Not to scale
Available Tractive Effort

- Force available to overcome resistance and accelerate a vehicle. There are two ways to achieve this:
  - Vehicle’s engine
  - Maximum value that will be a function of the vehicle’s weight distribution and the characteristics of the road surface.

Sign indicating that vehicle should hit breaks since it is approaching a downhill.

www2.irdinc.com
Maximum Tractive Effort

- No matter how much force a vehicle’s engine produces, there is a point beyond where this force does not overcome resistance or accelerates the vehicle.

Figure 2.3 Vehicle forces and moment-generating distances.
Maximum Tractive Effort

**Figure 2.3** Vehicle forces and moment-generating distances.

- \( F_f \) = available tractive effort of the front tires in lb (N),
- \( F_r \) = available tractive effort of the rear tires in lb (N),
- \( W \) = total vehicle weight in lb (N),
- \( W_f \) = weight of the vehicle on the front axle in lb (N),
- \( W_r \) = weight of the vehicle on the rear axle in lb (N),
- \( \Theta_g \) = angle of the grade in degrees,
- \( m \) = vehicle mass in slugs (kg),
- \( a \) = rate of acceleration in ft/s\(^2\) (m/s\(^2\)),
- \( L \) = length of wheelbase,
Maximum Tractive Effort

Cont. Figure 2.3

- $h =$ height of the center of gravity above the roadway surface,
- $l_f =$ distance from the front axle to the center of gravity, and
- $l_r =$ distance from the rear axle to the center of gravity.

Adding moments to determine maximum tractive effort:

$$W = \left[ R_a \times h + W_{lf} \times \cos \Theta_g + m \times a \times h \pm W \times h \times \sin \Theta_g \right]$$

$$\text{-------------------}$$

L

*** $\{W \times h \times \sin \Theta g\}$ is positive for upward slopes and negative for downward slopes
Maximum Tractive Effort

- Rearranging terms and using basic physics, the maximum tractive effort will be the normal force multiplied by the coefficient of road adhesion.

  - For Rear wheel drive:
    \[
    F_{\text{max}} = \frac{\mu W (l_f - f_{rl} h)/L}{1 - \mu h/L}
    \]

  - For Front wheel drive:
    \[
    F_{\text{max}} = \frac{\mu W (l_r + f_{rl} h)/L}{1 + \mu h/L}
    \]

\( \mu \) = coefficient of road adhesion
In some instances, the coefficient of road adhesion values can exceed the value of 1.

<table>
<thead>
<tr>
<th>Pavement</th>
<th>Coefficient of road adhesion</th>
<th>Maximum</th>
<th>Slide</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good, dry</td>
<td></td>
<td>1.00*</td>
<td>0.80</td>
</tr>
<tr>
<td>Good, wet</td>
<td></td>
<td>0.90</td>
<td>0.60</td>
</tr>
<tr>
<td>Poor, dry</td>
<td></td>
<td>0.80</td>
<td>0.55</td>
</tr>
<tr>
<td>Poor, wet</td>
<td></td>
<td>0.60</td>
<td>0.30</td>
</tr>
<tr>
<td>Packed snow or ice</td>
<td></td>
<td>0.25</td>
<td>0.10</td>
</tr>
</tbody>
</table>

*In some instances, the coefficient of road adhesion values can exceed 1.0. See discussion at the end of Section 2.7.

Example 2.3

A 2500-lb (11.1 kN) car is designed with a 120-inch (3.048-m) wheelbase. The center of gravity is located 22 inches (559 mm) above the pavement and 40 inches (1.12 m) behind the front axle. If the coefficient of road adhesion is 0.6, what is the maximum tractive effort that can be developed if the car is (a) front-wheel drive and (b) rear wheel drive?
Engine Generated Tractive Effort

- The two most commonly used measures for engine output are:
  - Torque – work generated by the engine. (ft-lb or N-m)
  - Power – rate of engine work, expressed in horsepower (hp or kW).
- Power is related to torque by the following equations:

\[
\begin{align*}
\text{hp}_e &= \frac{2\pi M_e n_e}{550} \\
\text{P}_e &= \frac{2\pi M_e n_e}{1000}
\end{align*}
\]

(2.16)
Engine Generated Tractive Effort

Where:

- $h_{pe}$ = engine-generated horsepower (1 horsepower equals 550 ft-lb/s),
- $P_e$ = engine-generated power in kW,
- $M_e$ = engine torque in ft-lb (Nm), and
- $n_e$ = engine speed in revolutions per second (the speed of the crankshaft).

Figure 2.4 Typical torque-power curves for a gasoline powered automobile engine.
Engine Generated Tractive Effort

- Gear reduction provides the mechanical advantage necessary for acceptable vehicle acceleration.
- There are two factors that determine the amount of tractive effort reaching the drive wheels:
  - Mechanical efficiency of the driveline ($n_d = 0.75$ to $0.95$)
  - Overall gear reduction ratio

\[
F_e = \frac{M_e \varepsilon_0 n_d}{r}
\]  

Engine Generated Tractive Effort (2.17)
Engine Generated Tractive Effort

Where:

- $F_e$ = engine-generated tractive effort reaching the driving wheels in lb (N),
- $r$ = radius of the drive wheels in ft (m),
- $M_e$ = engine torque in ft-lb (Nm),
- $\varepsilon_0$ = overall gear reduction ratio, and
- $\eta_d$ = mechanical efficiency of the driveline (0.75 to 0.95).
Engine Generated Tractive Effort

- Relationship between vehicle speed and engine speed is:

\[ V = \frac{2\pi r n_e (1 - i)}{\epsilon_0} \]  

(2.18)

Where:

- \( V \) = vehicle speed in ft/s (m/s),
- \( n_e \) = crankshaft revolutions per second,
- \( i \) = slippage of the driveline, generally taken as 2 to 5% \((i = 0.02\text{ to } 0.05)\) for passenger cars, and
- Other terms as defined previously.
Engine Generated Tractive Effort

Example 2.4

It is known that an experimental engine has a torque curve of the form \( M_e = a n_e - b n_e^2 \), where \( M_e \) is engine torque in ft-lb, \( n_e \) is engine speed in revolutions per second, and \( a \) and \( b \) are unknown parameters. If the engine develops a maximum torque of 92 ft-lb (124.75 N-m) at 3200 rev/min, what is the engine’s maximum power?
Vehicle Acceleration

- There is a relationship between traction, resistance, and forces available to accelerate.

- Basic equation: \[ F - \sum R = \gamma_m ma \] (2.19)
  - \( F - \sum R \) refers to force available to accelerate.
  - \( \gamma_m \) is the mass factor (rotational inertia).
  - This is approximated as \( \gamma_m = 1.04 + 0.0025\varepsilon_0^2 \) (2.20)

- This is an adaptation of equation 2.2, taking note that the vehicle’s rotating parts provide an inertia which must be overcome for acceleration.
Vehicle Acceleration

- The force required to accelerate is \( F_{\text{net}} = F - \sum R \).
  - If \( F_{\text{net}} = 0 \), the vehicle is already at maximum speed.
  - If \( F_{\text{net}} > 0 \), the vehicle is traveling at less than maximum speed.

- The two measures of vehicle acceleration that we study in depth are time and distance to accelerate.
  - For the case of \( F_{\text{net}} > 0 \), the equation can be written in differential form as:
    \[ F_{\text{net}} = \gamma m \frac{dV}{dt} \text{ or } dt = \frac{\gamma m dV}{F_{\text{net}}} \]
Vehicle Acceleration

- As $F_{\text{net}}$ is a function of vehicle speed, $F_{\text{net}} = f(V)$

- This relationship is used to integrate $F_{\text{net}}$
  - Time to accelerate: $t = \gamma_m m \int_{v_1}^{v_2} \frac{dV}{f(V)}$ (2.21)
  - Distance to accelerate: $d_a = \gamma_m m \int_{v_1}^{v_2} \frac{VdV}{f(V)}$
Vehicle Acceleration

Example 2.5

A car is traveling at 10 mi/h (16.1 km/h) on a roadway covered with hard-packed snow (coefficient of road adhesion of 0.2). The car has $C_d = 0.3$, $A_f = 20$ ft$^2$ (1.86 m$^2$), and $W = 3000$ lb (13.34kN). The wheelbase is 120 inches (3.048 m), and the center of gravity is 20 inches (508 mm) above the roadway surface and 50 inches (1.27 m) behind the front axle. The air density is 0.002045 slugs/ft$^3$ (1.054 kg/m$^3$). The car’s engine is producing 95 ft-lb (128.8 N-m) of torque and is in a gear that gives an overall gear reduction ratio of 4.5 to 1, the wheel radius is 14 inches (356 mm), and the mechanical efficiency of the driveline is 80%. If the driver needs to accelerate quickly to avoid an accident, what would the acceleration be if the car is (a) front-wheel drive and (b) rear wheel drive?
Fuel Efficiency

- Several factors influence a vehicle’s fuel efficiency:
  - Engine design
  - Driveline slippage
  - Driveline efficiency
  - Aerodynamics
  - Frontal area
  - Weight
  - Tire design

Principles of Braking

- In order to understand braking principles, first we consider the force and moment-generating diagram.

**Figure 2.7** Forces acting on a vehicle during braking, with driveline resistance ignored.
Principles of Braking

- **Figure 2.7** Forces acting on a vehicle during braking, with driveline resistance ignored
  - $R_a = \text{aerodynamic resistance in lb (N)}$
  - $R_{lf} = \text{rolling resistance of the front tires in lb (N)}$
  - $R_{rlr} = \text{rolling resistance of the rear tires in lb (N)}$
  - $F_{bf} = \text{braking force on the front tires in lb (N)}$
  - $F_{br} = \text{braking force on the rear tires in lb (N)}$
  - $W = \text{total vehicle weight in lb (N)}$
  - $W_f = \text{weight of the vehicle on the front axle in lb (N)}$
  - $W_r = \text{weight of the vehicle on the rear axle in lb (N)}$
Principles of Braking

- Figure 2.7 continued
  - \( m \) = vehicle mass in slugs (kg)
  - \( a \) = acceleration in ft/s\(^2\) (m/s\(^2\))
  - \( L \) = length of wheelbase
  - \( h \) = height of the center of gravity from the road surface
  - \( l_f \) = distance from the front axle to the center of gravity
  - \( l_r \) = distance from the rear axle to the center of gravity
  - \( \theta_g \) = angle of the grade in degrees
Principles of Braking

- When a vehicle brakes, there is a load transfer from the rear to the front axle.
- The normal loads experienced by the front and rear axles can be expressed by summing the moments about contact points between the tires and road.

Front:

\[
W_f = \frac{1}{L} \left[ Wl_r + h (ma - Ra \pm W \sin \theta_g) \right]
\]  \hspace{1cm} (2.23)

Rear:

\[
W_r = \frac{1}{L} \left[ Wl_f - h (ma - Ra \pm W \sin \theta_g) \right]
\]  \hspace{1cm} (2.24)
Principles of Braking

- The normal force equations can be rewritten by summing the forces along the vehicle’s longitudinal axis:

\[ F_b + f_{rl}W = ma - R_a \pm W \sin \theta_g \]  

(2.25)

- From equation 2.6, we know that

\[ R_{rl} = f_{rl}W \]

\[ F_b = F_{bf} + F_{br} \]

- We also have the relationship of

- The above equations are substituted in 2.23, 2.24:

- Front:

\[ W_f = \frac{1}{L} \left[ W l_r + h(F_b + f_{rl}W) \right] \]  

(2.26)

- Rear:

\[ W_r = \frac{1}{L} \left[ W l_f - h(F_b + f_{rl}W) \right] \]  

(2.27)
Principles of Braking

The maximum vehicle braking force \( F_{\text{b max}} \) is related to the vehicle’s weight and coefficient of road adhesion \( \mu \).

- **Front:**
  \[
  F_{bf_{\text{max}}} = \mu W_f
  \]
  and by substitution:
  \[
  F_{bf_{\text{max}}} = \frac{\mu W}{L} [l_r + h(\mu + f_{rl})] \quad (2.28)
  \]

- **Rear:**
  \[
  F_{br_{\text{max}}} = \mu W_r
  \]
  and by substitution:
  \[
  F_{br_{\text{max}}} = \frac{\mu W}{L} [l_f + h(\mu + f_{rl})] \quad (2.29)
  \]
Principles of Braking

- The tires should be at a point of impending slide in order for maximum braking forces to develop.
- Braking forces will decline when the wheels are locked, which results from reduced road adhesion. The extent of this reduction depends on the pavement and weather conditions (Table 2.4).

<table>
<thead>
<tr>
<th>Pavement</th>
<th>Coefficient of road adhesion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good, dry</td>
<td>Maximum: 1.00* Slide: 0.80</td>
</tr>
<tr>
<td>Good, wet</td>
<td>0.90</td>
</tr>
<tr>
<td>Poor, dry</td>
<td>0.80</td>
</tr>
<tr>
<td>Poor, wet</td>
<td>0.60</td>
</tr>
<tr>
<td>Packed snow or ice</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Principles of Braking

- The maximum attainable vehicle deceleration from the vehicle’s braking system is equal to $\mu * g$, where $\mu$ is the coefficient of road adhesion and $g$ is the gravitational constant.

- Vehicle braking systems must distribute the braking forces between the front and rear brakes, which is typically done through hydraulic pressures.

- The front/rear proportioning of these braking forces will be optimal when it is the same proportion as the ratio of maximum braking forces on the front and rear axles.
Principles of Braking

- Maximum braking forces will be developed when the brake force ratio is

\[ BFR_{f/r_{\text{max}}} = \frac{l_r + h(\mu + f_{rl})}{l_f - h(\mu + f_{rl})} \]  

- \( BFR_{f/r_{\text{max}}} \) = the brake force ratio, allocated by the vehicle’s braking system, that results in maximum (optimal) braking forces

- The percentages of braking force that should be allocated to the axles are as follows:
  - Front:
    \[ PBF_f = 100 - \frac{100}{1 + BFR_{f/r_{\text{max}}}} \]  
  - Rear:
    \[ PBF_r = \frac{100}{1 + BFR_{f/r_{\text{max}}}} \]
Principles of Braking

Example 2.6
A car has a wheelbase of 100 inches (2.54 m) and a center of gravity that is 40 inches (1.016 m) behind the front axle at a height of 24 inches (609 mm). If the car is traveling at 80 mi/h (128.7 km/h) on a road with poor pavement that is wet, determine the percentages of braking force that should be allocated to the front and rear brakes to ensure that maximum braking forces are developed.
Principles of Braking

- The optimal brake force proportions will be altered with the addition of passengers and cargo, as there is now additional loading. This shifts the weight distribution and height of the center of gravity.
- True optimal brake force proportioning is rarely achieved in non-antilock braking systems, so a braking efficiency term is defined: $\eta_b = \frac{g_{\text{max}}}{\mu}$ (2.33)
  - $\eta_b = \text{braking efficiency}$
  - $g_{\text{max}} = \text{maximum deceleration in g units}$
  - $\mu = \text{coefficient of road adhesion}$
Principles of Braking

- **Figure 2.8** highlights the effects of various vehicle loadings on the distribution of braking forces.

![Diagram showing the effect of brake force proportioning on braking performance of a light truck and a passenger car.](image)
Principles of Braking

- In order to prevent the wheels from locking during a braking application, many cars are now equipped with anti-lock braking systems (ABS).
  - ABS prevents the coefficient of road adhesion from dropping to slide values
  - They also have the potential to raise braking efficiency to 100%

- ABS vs. non-ABS: [http://www.youtube.com/watch?v=-Zv3Sacl7JQ&feature=related](http://www.youtube.com/watch?v=-Zv3Sacl7JQ&feature=related)
Principles of Braking

The relationship between the stopping distance, braking force, vehicle mass, and vehicle speed is

\[ ads = \left[ \frac{F_b + \sum R}{\gamma_b m} \right] ds = VdV \quad (2.34) \]

\[ \gamma_b = \text{mass factor accounting for moments of inertia during braking, which is 1.04 for automobiles} \]

We must integrate to find the stopping distance:

\[ S = \int_{v_1}^{v_2} \gamma_b m \frac{VdV}{F_b + \sum R} \quad (2.35) \]
Principles of Braking

- By substituting resistances in equation 2.35:

\[
S = \gamma_b m \int_{v_1}^{v_2} \frac{VdV}{F_b + R_a + f_{rl}W \pm W \sin \theta_g}
\]

- \(V_1\) = initial vehicle speed in ft/s (m/s)
- \(V_2\) = final vehicle speed in ft/s (m/s)
- \(f_{rl}W\) = rolling resistance
- \(W\sin\theta_g\) = grade resistance (+ for uphill, - for downhill)
Principles of Braking

- By making some assumptions and integrating equation 2.36, we get

\[
S = \frac{\gamma_b W}{2gK_a} \ln \left[ \frac{\mu W + K_a V_1^2 + f_{rl} W \pm W \sin \theta_g}{\mu W + K_a V_2^2 + f_{rl} W \pm W \sin \theta_g} \right]
\]

(2.39)

- We assume that the effect of speed on \(f_{rl}\) is constant and can be approximated using the average velocity
- We also let \(m = W/g\) and \(F_b = \mu W\)
- The equation also utilizes the simplification

\[
K_a = \frac{\rho}{2} C_D A_f
\]
Principles of Braking

- If braking efficiency is considered, the actual braking force is \( F_b = \eta_b \mu W \) \hspace{1cm} (2.41)
- Therefore, the theoretical stopping distance is:

\[
S = \frac{\gamma_b W}{2gK_a} \ln \left[ 1 + \frac{K_a V_1^2}{\eta_b \mu W + f_{rl} W \pm W \sin \theta_g} \right]
\]

\hspace{1cm} (2.42)

- Equation 2.35 can also be integrated to give an alternative equation for theoretical stopping distance

\[
S = \frac{\gamma_b (V_1^2 - V_2^2)}{2g(\eta_b \mu + f_{rl} \pm \sin \theta_g)}
\]

\hspace{1cm} (2.43)

- Note: aerodynamic resistance is ignored
Principles of Braking

Example 2.7

A new experimental 2500-lb (11.12 kN) car, with $C_d = 0.25$ and $A_f = 18 \text{ ft}^2 (1.67 \text{ m}^2)$, is traveling at 90 mi/h (144.8 Km/h) down a 10% grade. The coefficient of road adhesion is 0.7 and the air density is 0.0024 slugs.$\text{ft}^3$ (1.24 kg/$\text{m}^3$). The car has an advanced antilock braking system that gives it a braking efficiency of 100%. Determine the theoretical minimum stopping distance for the case where aerodynamic resistance is considered and the case where aerodynamic resistance is ignored.
The previous discussion on vehicle theoretical stopping distances did not take into account the driver’s reaction time, among other factors.

The basic physics equation of \( V_2^2 = V_1^2 + 2ad \) serves as the basis for practical stopping distance.

The equation is rearranged to:

\[
d = \frac{V_1^2 - V_2^2}{2a}
\]

If the vehicle comes to a complete stop, that is \( V_2 = 0 \), then the equation becomes:

\[
d = \frac{V_1^2}{2a}
\]

AASHTO recommends a deceleration rate, \( a \), of 11.2 ft/s\(^2\) (3.2 m/s\(^2\)).
Principles of Braking

- Equation 2.46 is modified to account for the effect of grade:
  \[ d = \frac{V_1^2}{2g\left(\frac{a}{g} \pm G\right)} \]  
  \( (2.47) \)

- By assuming that the vehicle comes to a complete stop, that \( \sin \theta_g = \tan \theta_g = G \) for small grades, and that \( \gamma_b \) and \( f_{rl} \) can be ignored, we have
  \[ S = \frac{V_1^2}{2g(\eta_b\mu \pm G)} \]  
  \( (2.48) \)
Principles of Braking

- In order to ensure that there is sufficient sight distance for a driver to safely stop, the distance traveled during driver perception/reaction must be taken into consideration:

\[ d_r = V_1 \times t_r \]  

(2.49)

- \( V_1 \) = initial vehicle speed in ft/s (m/s)
- \( t_r \) = time required to perceive/react to the need to stop

- The perception/reaction time is influenced by several factors, such as driver’s age and emotional state, so AASHTO recommends a conservative estimate of 2.5 seconds for calculation purposes.
Principles of Braking

- We can now see that the overall required stopping distance is a combination of theoretical and practical braking distances, as well as the distance traveled during perception/reaction.

- This is summed up as \[ d_s = d + d_r \] (2.50)

- \( d_s \) = total stopping distance in ft (m)
- \( d \) = distance traveled during braking in ft (m)
- \( d_r \) = distance traveled during perception/reaction in ft (m)
Principles of Braking

Example 2.10

Two drivers each have a reaction time of 2.5 seconds. One is obeying a 55-mi/h (88.5-km/h) speed limit, and the other is traveling illegally at 70 mi/h (112.6 km/h). How much distance will each of the drivers cover while perceiving/reacting to the need to stop, and what will the total stopping distance be for each driver (using practical stopping distance assuming $G = -2.5\%$)